Name of College - S.S. College J. Bad.
Deptt - Mathematics
Topic - Adjugate and inverse of a Matrix (Contd)
Class - B.Sc(Hons)
Time - ~~[illegible]~~ 1:00 P.M To 1.45 P.M
Date - 22-09-2020

By - Samrendra kumar.

1. Show that the adjoint of a scalar matrix is a scalar matrix

Proof :→ Let $A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}_{3\times3}$ be a scalar matrix

$A_{11} = \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} = k^2$ $\quad A_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & k \end{vmatrix} = 0$ $\quad A_{13} = \begin{vmatrix} 0 & k \\ 0 & 0 \end{vmatrix} = 0$

$A_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & k \end{vmatrix} = 0$ $\quad A_{22} = \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} = k^2$ $\quad A_{23} = -\begin{vmatrix} k & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = \begin{vmatrix} 0 & 0 \\ k & 0 \end{vmatrix} = 0$ $\quad A_{32} = -\begin{vmatrix} k & 0 \\ 0 & 0 \end{vmatrix} = 0$ $\quad A_{33} = \begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix}$

$$\operatorname{adj} A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{bmatrix}^T = \begin{bmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{bmatrix} = [k^2]$$

Show that
If $A = [a_{ij}]$ be a square Matrix of order n, show that

(i) $adj\, A^T = (adj\, A)^T$

(ii) $adj\, A^* = (adj\, A)^*$

(iii) adj of a symmetric Matrix is a symmetric Matrix

(iv) adj of Hermitian Matrix is a Hermitian Matrix

(1) for $adj\, A^T = (adj\, A)^T$

Let $A = [a_{ij}]$ be a square Matrix of order n and A^T is the Transpose of A.

Since $|A| = |A^T|$ ——— (i)

$A \cdot (adj\, A) = (adj\, A) \cdot A = |A| I$ ——— (ii)

On Replacing A by A^T in (ii)

$A^T (adj\, A^T) = (adj\, A^T) A^T = |A^T| I$ but $|A| = |A^T|$
$= |A| I$ ——— (iii)

On Taking Transpose on both sides (iii)

$$\left[A^T (adj\, A^T)\right]^T = \left[(adj\, A^T) A^T\right]^T$$

on Applying Reversal Rule.

$(adj\, A^T)^T (A^T)^T = (A^T)^T (adj\, A^T)^T$
$(adj\, A^T)^T A = A\, (adj\, A^T)^T = |A| I$ ——— (iv)

On comparing (ii) and (iv)

we get $(adj A^T)^T = adj A$.

Again Taking Transpose on both sides

$$[(adj A^T)^T]^T = (adj A)^T$$

$$\Rightarrow adj A^T = (adj A)^T.$$

Proved.

(ii) Show that $adj A^* = (adj A)^*$ — (i)

We have $adj(A^*) = adj(\bar{A})^T$

$= adj\,(\bar{a}_{ij}]^T$

$= adj\,(\bar{a}_{ji})$

$\therefore\ adj(A^*) = (\bar{A}_{ji})^T = (\bar{A}_{ij})^T$ — (ii)

Again $adj A = [A_{ij}]^T$

$= A_{ij}^T$

$= A_{ji}$

$(adj A)^* = (A_{ji})^*$

$= (\bar{A}_{ji})^T$

$= [\bar{A}_{ij}]$ — (iii)

From (ii) and (ii)

$(adj A)^* = (adj A^*)$

(iii) We are to show adjoint of a Symmetric Matrix is Symmetric Matrix (4)

Let $A = [a_{ij}]$ be a Symmetric Matrix

$\Rightarrow A^T = A$.

$\therefore \text{adj} A^T = (\text{adj} A)^T$

$\Rightarrow (\text{adj} A)^T = \text{adj} A^T \qquad \because A^T = A$

$= \text{adj} A$.

$\Rightarrow (\text{adj} A)^T = \text{adj} A \Rightarrow$ adj A is Symmetric Matrix

Thus adjoint of Symmetric Matrix is Symmetric Matrix

(iv) Let $A = [a_{ij}]$ be a Hermitian matrix

$\Rightarrow A^* = A$.

$\Rightarrow \text{adj}(A^*) = (\text{adj} A)^*$

$\Rightarrow \text{adj} A = (\text{adj} A)^* \qquad \because A^* = A$.

Thus $\text{adj}(A^*) = \text{adj} A \Rightarrow$

adjoint of Hermitian Matrix is Hermitian.